Problem Sheet Unit 1: Exp Met. & Ana

Binomial, Poisson & Normal distribution

Binomial Distribution Problems

1. A production process for engine parts has a 95% success rate. If 20 parts are manufactured, what is the probability that:
   (i) Exactly 18 parts will be produced successfully?
   (ii) Between 15 to 18 parts will be produced successfully?
   (iii) More than 17 parts will be produced successfully?
2. A mechanical system is designed to have a 90% reliability rate. If 40 systems are tested, what is the probability that:
   (i) Exactly 36 systems will operate reliably?
   (ii) Not more than 30 systems will operate reliably?
3. In a batch of 50 bolts, there is a 5% chance of each bolt being defective. What is the probability that 3 or fewer bolts will be defective?
4. In a study, it is found that 20% of machines require maintenance within the first year. If 30 machines are installed, what is the probability that between 4 and 7 machines will require maintenance within the first year?
5. A new fuel injector design has a 90% success rate in performance tests. If 30 fuel injectors are tested, what is the probability that:
   (i) Exactly 27 injectors will pass the test?
   (ii) Between 25 to 28 injectors will pass the test?
   (iii) More than 26 injectors will pass the test?
6. The reliability of a braking system is 92%. If 50 braking systems are tested, what is the probability that:
   (i) Exactly 46 systems will be reliable?
   (ii) Not more than 40 systems will be reliable?
7. In a batch of 100 tires, there is a 3% chance of each tire being defective. What is the probability that 4 or fewer tires will be defective?
8. It is found that 10% of vehicles require a software update within the first year. If 20 vehicles are sold, what is the probability that between 2 and 5 vehicles will require the update?
9. A manufacturing process has a 95% success rate in producing defect-free products. If 50 products are made, what is the probability that:
   (i) Exactly 47 products will be defect-free?
   (ii) Between 45 to 48 products will be defect-free?
   (iii) More than 46 products will be defect-free?
10. The reliability of a new product is 85%. If 20 products are tested:
    (i) What is the probability that exactly 17 products will be reliable, and at least 18 products will be reliable?
    (ii) What is the probability that no more than 15 products will be reliable? Also, find the cumulative probability for 12 to 15 reliable products.

Poisson Distribution Problems

6. A factory experiences an average of 2 equipment failures per week. What is the probability that exactly 3 equipment failures will occur in the next two weeks?
7. A specific type of fault in a machine occurs at an average rate of 1 per month. What is the probability that no faults will be reported in a month?
8. An automobile part experiences an average of 4 failures in 20 days. What is the probability of observing 6 failures in the next 40 days?

9. A specific defect occurs at an average rate of 2 per day in a production line. What is the probability that no defects will be reported in a day?
10. An assembly line produces an average of 1 defective unit per hour. What is the probability that in a given hour, there will be exactly 2 defective units?
11. A machine experiences an average of 2 breakdowns in 10 days. What is the probability of observing 3 breakdowns in the next 20 days?
12. In a batch of 1000 units, there is a 0.004% chance of each unit being defective:
    (i) What is the probability that 5 or fewer units will be defective?
    (ii) What is the probability that more than 8 but fewer than 12 units will be defective?

Normal Distribution Problems

11. The tensile strength of a certain material is normally distributed with a mean of 500 MPa and a standard deviation of 50 MPa. What is the probability that a randomly selected sample has a tensile strength above 550 MPa?
12. A researcher studies the weight of a new material and finds that it follows a normal distribution with a mean of 200 grams and a standard deviation of 10 grams.
    (i) What is the probability that a sample weighs more than 220 grams?
    (ii) What could be the maximum weight for the lightest 10% of samples?
13. The thickness of metal sheets produced by a machine is normally distributed with a mean of 0.5 cm and a standard deviation of 0.05 cm. What is the probability that a randomly selected sheet has a thickness between 0.45 cm and 0.55 cm?
14. The lifespan of a certain type of bearing is normally distributed with a mean of 1000 hours and a standard deviation of 100 hours.
    (i) What is the probability that a bearing lasts less than 900 hours?
    (ii) What is the maximum possible lifespan for 95% of the bearings?
11. A researcher studies the weight of a new car part and finds that it follows a normal distribution with a mean of 15 kg and a standard deviation of 1 kg.
    (i) What is the probability that a part weighs more than 16.5 kg?
    (ii) What could be the maximum weight for the lightest 5% of parts?
13. The speed of cars passing through a toll booth is normally distributed with a mean of 60 mph and a standard deviation of 5 mph. What is the probability that a randomly selected car is traveling between 55 mph and 65 mph?
14. The lifespan of car batteries is normally distributed with a mean of 4 years and a standard deviation of 0.5 years.
    (i) What is the probability that a battery lasts less than 3.5 years?
    (ii) What is the maximum possible lifespan for 90% of the batteries?
15. The time taken to assemble an engine is normally distributed with a mean of 2 hours and a standard deviation of 0.3 hours. What is the probability that a randomly selected assembly takes longer than 2.5 hours?
16. The tensile strength of a certain car part follows a normal distribution with a mean of 1000 N and a standard deviation of 50 N. What is the probability that a randomly selected part can withstand more than 1050 N?
17. The tensile strength of a certain material is normally distributed with a mean of 500 MPa and a standard deviation of 50 MPa.
    (i) What tensile strength value separates the top 10% of the material from the rest?
    (ii) What tensile strength value corresponds to the 25th percentile?
18. The lifespan of a certain type of bearing is normally distributed with a mean of 1000 hours and a standard deviation of 100 hours.
    (i) What lifespan value corresponds to the bottom 5% of the bearings?
    (ii) What lifespan value separates the middle 50% of the bearings from the rest?

19. The diameter of manufactured metal rods is normally distributed with a mean of 10 cm and a standard deviation of 0.5 cm.
    (i) What diameter value separates the smallest 20% of the rods from the rest?
    (ii) What is the diameter that exceeds 90% of the rods?
20. The breaking strength of a new material is normally distributed with a mean of 600 N and a standard deviation of 30 N.
    (i) What breaking strength value corresponds to the 95th percentile?
    (ii) What is the minimum breaking strength value for the strongest 10% of the material?